FINDING THE EQUATION OF A LINE USING $y - y_1 = m(x - x_1)$

The slope-intercept form of a line, $y = mx + b$, is perfect when you have the slope and the y-intercept. But the odds of this are slim. It's more likely that you'll be working with the slope and some point on the line other than the y-intercept. So, in the following formula, m is the slope (as before), while (x_1, y_1) represents any given point on the line.

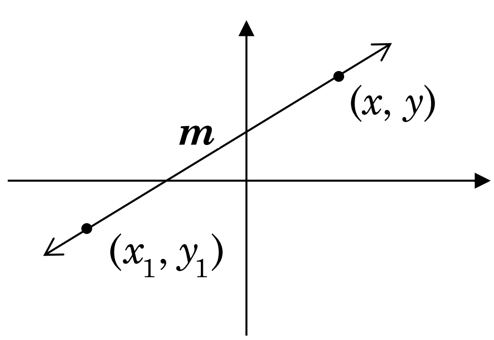
❑ *THE POINT-SLOPE FORMULA*

THEOREM: **The equation of the line with slope m and passing through the point (x_1, y_1) is**

$$y - y_1 = m(x - x_1)$$

The Point-Slope Form of a Line

PROOF: We begin by sketching the line and labeling the given point (x_1, y_1), the slope m, and a *generic point* (x, y):



The Point-Slope Form of a Line

On the one hand, the slope of the line is given by m. On the other hand, the slope of the line can be calculated using the two points (x, y) and (x_1, y_1): $\frac{y - y_1}{x - x_1}$. And, of course, these two slopes must be the same (since there's only one line involved):

$$\frac{y - y_1}{x - x_1} = m$$

$$\Rightarrow \quad \boxed{y - y_1 = m(x - x_1)}$$

Q.E.D.

❑ Bonus Derivation

Using the point-slope form of a line above, we can derive the slope-intercept form of a line: $y = mx + b$. Here's how:

We assume that the slope of a line is given by m, and that the y-intercept is $(0, b)$, which is simply a point on the line; so it's the point (x_1, y_1) in the formula $y - y_1 = m(x - x_1)$. Thus,

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow \quad y - b = m(x - 0)$$

$$\Rightarrow \quad y - b = mx$$

$$\Rightarrow \quad y = mx + b$$

and done!

❑ *EXAMPLES OF THE POINT-SLOPE FORMULA*

EXAMPLE 1: **Find the equation of the line whose slope is –3 and which passes through the point (8, –2).**

Solution: This is precisely the data we need to use the point-slope form: $y - y_1 = m(x - x_1)$. We're given the slope, so $m = -3$. We're also given a point on the line, so $(x_1, y_1) = (8, -2)$. Plugging these values into the point-slope form gives us

$$y - (-2) = -3(x - 8), \text{ or}$$

$$\boxed{y + 2 = -3(x - 8)}$$

EXAMPLE 2: **Find the equation of the line passing through the two points (3, –5) and (–2, –8).**

Solution: The point-slope form, $y - y_1 = m(x - x_1)$, requires a point (we have two of them), and the slope, which we'll have to calculate ourselves:

$$m = \frac{\Delta y}{\Delta x} = \frac{-5-(-8)}{3-(-2)} = \frac{-5+8}{3+2} = \frac{3}{5}$$

Now, using the point (3, –5) (either point would work), we get our equation

$$y - (-5) = \tfrac{3}{5}(x - 3), \text{ or } \boxed{y + 5 = \tfrac{3}{5}(x - 3)}$$

Homework

1. Use the point-slope formula to find the equation of the line with slope 7 and passing through the point (6, –8).

2. Use the point-slope formula to find the equation of the line with slope 0 and passing through the point (–17, 9).

3. Use the point-slope formula to find the equation of the line with slope $-\frac{4}{7}$ and passing through the point $(\frac{1}{2}, \pi)$.

4. Use the point-slope formula to find the equation of the line which passes through the points (–2, 4) and (5, –5).

5. Use the point-slope formula to find the equation of the line which passes through the points $(\pi, \sqrt{2})$ and (–3, 1).

Solutions

1. $y + 8 = 7(x - 6)$

2. $y - 9 = 0$

3. $y - \pi = -\frac{4}{7}\left(x - \frac{1}{2}\right)$

4. $y - 4 = -\frac{9}{7}(x + 2)$

5. $y - \sqrt{2} = \frac{1 - \sqrt{2}}{-3 - \pi}(x - \pi)$ The slope can also be written: $\frac{\sqrt{2} - 1}{\pi + 3}$.

An investment in knowledge always pays the best interest.

Benjamin Franklin